

AD-A088 558

SOUTHEASTERN MASSACHUSETTS UNIV NORTH DARTMOUTH DEPT --ETC F/6 9/4  
STUDY OF A CLASS OF NON-GAUSSIAN SIGNAL PROCESSING PROBLEMS. (U)

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N00014-79-C-0494

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SMU-EE-TR-80-6

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**LEVEL 4**

Technical Report  
Contract Number N00014-79-C-0494  
(15) SMU-EE-TR-80-6  
August 12, 1980

(12)

AD A088558

(11) SMU-EE-TR-80-6

(6)

STUDY OF A CLASS OF  
NON-GAUSSIAN SIGNAL PROCESSING PROBLEMS\*

(9) Technical Report

(10)

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JUL 2 1980

\*The support of the Statistics and Probability Program of the Office of Naval Research on this work is gratefully acknowledged.

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C. H. CHEN

Gaussian assumption has been a fundamental one in most statistical signal processing work. The assumption not only simplifies the analytical problems involved but also matches the data characteristics in many cases because of the law of large numbers. In a number of Navy sonar, radar and communications systems, signal processing algorithms must be developed without the Gaussian assumption.

The need for non-Gaussian signal processing

1. To investigate non-linear adaptive procedures to assess their detection performance against transient signals.
2. To investigate non-linear spectral analysis techniques such as the Maximum Entropy Spectral Analysis (MESA) and the Maximum Likelihood Method (MLM) to assess their detection performance against transient signals.
3. To investigate techniques for weak signal extraction in the presence of strong interfering signals. As a special case, potential techniques for detecting broad-band (such as spread-spectrum) signals can be explored including the analysis of changes in noise statistics caused by pseudo-noise signals.
4. To explore the use of nonparametric and robust statistical approaches to devise algorithms which perform well under different noise characteristics.

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5. To study non-Gaussian models appropriate for reverberation or reverberation-like noise found in shallow water environment.
6. To examine the effects of non-Gaussian sources of impulsive noise interference in time-delay estimation, passive range estimation and related underwater acoustic signal processing problems.
7. To enlarge the basic mathematical and statistical theory of non-Gaussian stochastic processes applicable to both electromagnetic and underwater sound environments .
8. To characterise the non-Gaussian channels and interferences including the multipath, dispersion, doppler effects and the optimal reception under these circumstances.
9. To examine the modifications required in the existing signal processing system hardware for non-Gaussian signal processing.

Obviously good solutions to the above problems require expertise in several areas such as communications and information theory, pattern recognition, wave propagation, digital signal processing, image processing, and mathematical statistics, etc. As the problems are interrelated, they should be treated jointly as much as possible. Problems 1 and 2 are examined in detail in this report. For Problem 3, detection performance of spread-spectrum signals and the extraction of weak signals in Gaussian noise have been considered even though further work is much needed. For a general discussion of spread-spectrum systems, see [1]. Adaptive digital filtering and Kalman filtering are suitable techniques for extraction of broad-band non-Gaussian signals in noises. However analytical work on the detection performance is not available. For Problem 4, some existing analytical work on robust and nonparametric detection of signals in impulsive

noise and random interference (See [2] and the references listed in the paper) are useful. While general theoretical work based on general assumptions is very important, special cases should be considered so that the detection procedures and performances developed will match more closely to specific impulsive noise and random interference conditions considered. Proper mathematical modelling of impulsive noise, random interference and channels is thus required, which is the Problem 5 stated above.

Time-delay estimation and passive ranging estimation from an array of sensors are typical sonar signal processing problems (see e.g. [3] [4]). Problem 6 suggests that the non-Gaussian sources be taken into consideration as the existing work based on Gaussian assumption is quite restrictive. Problem 7 presents a number of mathematical and statistical topics to be explored. In fact some theoretical work for Gaussian signals and Gaussian noises can be extended to non-Gaussian cases [5]. For example, consider the detection and estimation of Gaussian signal in Gaussian noise. The conditional mean of the signal is the best estimate for a variety of criteria. In the non-Gaussian case it is the best linear estimate in the sense that mean-squared error is minimized. When the signal is unknown but non-random, the maximum likelihood estimate of the signal is the same whether the noise is Gaussian or non-Gaussian. The test statistics for detecting non-Gaussian signal in Gaussian noise is the same as that for detecting Gaussian signal in Gaussian noise at small signal condition. Many other mathematical properties remain to be examined.

Problem 8 has been well explored and reported in publications in communications, wave propagation and other areas. However, much less amount of work has been done for underwater transmission media. Although most signal processing

hardware designed for Gaussian signal processing can also be used for non-Gaussian signal processing with non-optimal performance, a modification of the hardware is a worthwhile effort to provide significant performance improvement in many cases.

## II. NON-LINEAR ADAPTIVE DETECTION PROCEDURES

One fundamental problem with the non-Gaussian signal processing is that the data is often nonstationary. Adaptive procedures are needed for estimation and detection of transient signals. Starting with the linear adaptive detection procedures, we have extensively explored the feasibility of Adaptive Digital Filtering and Kalman Adaptive Filtering [6] [7]. As the statistical characteristics of the data change with time, filter parameters must be re-adjusted at periodic intervals. This is considered as a piecewise linear adaptive procedure. If the data statistics are

monitored continuously and the parameters are adjusted with each significant change in data statistics, we will have a nonlinear adaptive procedure. Detection of the desired signal from the filtered output can be performed by the thresholding or correlation operation.

The Adaptive Digital Filter as shown in Figure 1 is a non-recursive digital noise by itself without making Gaussian assumption of the data. Let  $N$  be the order of the filter. The input-output relation of the filter is given by

$$g_m = \sum_{n=0}^N b_{n,m} f_{m-n} \quad (1)$$

where  $g_m$  is the filter output,  $b_{n,m}$  is the filter coefficient, and  $f_n$  is the filter output. Define the vectors

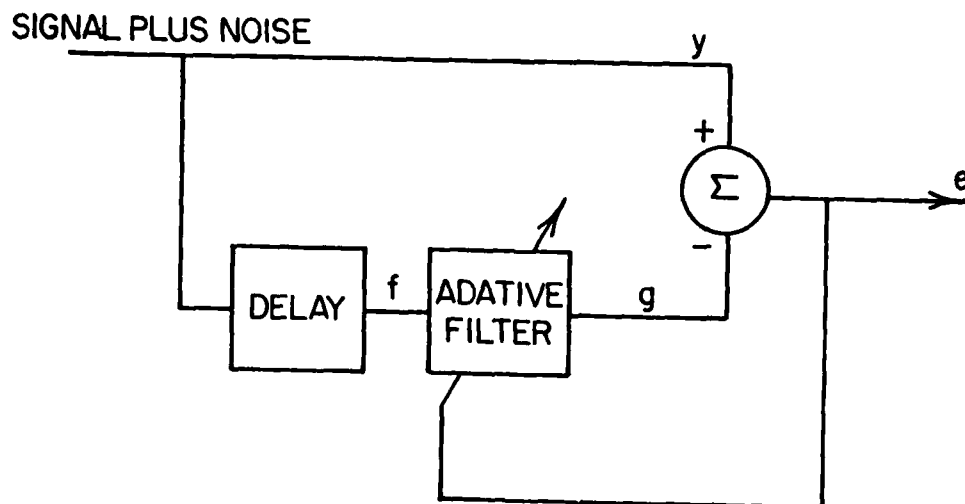


Fig. 1a The Adaptive Digital Filtering System.

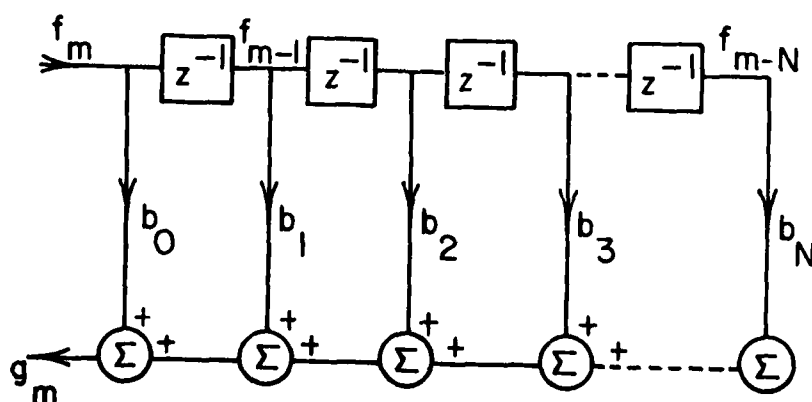


Fig. 1b The nonrecursive Adaptive Filter.

$$B_m' = [b_{0,m} \quad b_{1,m} \quad \dots \quad b_{N,m}]$$

and

$$F_m' = [f_m \quad f_{m-1} \quad \dots \quad f_{m-N}]$$

Equation (1) can be written as

$$q_m = B_m' F_m = F_m' B_m \quad (2)$$

By using the Noisy Least-Mean-Square Algorithm [8], the filter coefficients can be determined from the equations

$$\begin{cases} B_{m+1} = B_m + v e_m F_m \\ e_m = y_m - q_m \\ q_m = B_m' F_m \end{cases} \quad (3)$$

where  $v$  is a constant. After adjusting the parameters that include  $v$ ,  $N$ ,  $m$ , and delay time, we can obtain the desired result with a greatly improved signal-to-noise ratio. Obviously the Adaptive Digital Filter works well with stationary data. Parameter choice may require a lot of trial-and-error even though the filtering procedure is fairly simple.

The signal model of the Kalman Adaptive Filter is given by:

$$x_{k+1} = F_k x_k + w_k \quad (4)$$

where  $x_k$  is the state variable,  $w_k$  is the system noise,  $v_k$  is the measurement noise. Note that Gaussian assumption is not required of  $x_k$ ,  $v_k$  and  $w_k$ . Given a set of measurements  $z_k$ ,  $k = 1, 2, \dots, N$ , we would like to estimate  $x_k$ ,  $k = 1, 2, \dots, N$ , based on the state-space signal model given by Eq. (4). Since



$z_k = x_k + v_k$ , we have

$$z_{k+1} = x_{k+1} + v_{k+1} = (F_k x_k + w_k) + v_{k+1} \quad (5)$$

Define an error criterion function,

$$J_{k+1} = (z_{k+1} - (F_k x_k + w_k) - v_{k+1})^2 = L^2 \quad (6)$$

If  $z_{k+1}$  and  $x_k$  are known, we determine the values of  $w_k$ ,  $v_{k+1}$  and  $f_k$  iteratively such that the state space model is optimum in the minimum mean square error sense. The method of Steepest Descent can be used in the optimization process [9]. Once the optimum values of  $w_k$ ,  $v_{k+1}$ , and  $F_k$  are obtained, we can perform a one-step prediction of  $x_{k+1}$  by using the equation

$$x_{k+1} = F_k x_k + w_k \quad (7)$$

with  $w_k$  being the prediction error. The iterative procedure for the optimization at the  $i$ th iteration is given by

$$\begin{aligned} w_{k,i+1} &= w_{k,i} + \lambda(2L) \\ v_{k+1,i+1} &= v_{k+1,i} + \lambda(2L) \\ F_{k,i+1} &= F_{k,i} + \lambda(2Lx_k) \end{aligned} \quad (8)$$

Here the increment  $\lambda$  and  $w_{k,0}$ ,  $v_{k+1,0}$ , and  $F_{k,0}$  were given initially. In seismic data processing, it is reasonable to assume that

$$F_{k,0} = w_{k,0} = v_{k+1,0} = 0$$

Furthermore, the initial state can be chosen as  $x_0 = 0$ . These assumptions should be appropriate for many non-Gaussian signal processing problems.

It will be of interest to examine some computer results with the seismic data which is non-Gaussian [7]. Both Adaptive Digital Filtering and Kalman Adap-

tive Filtering can perform well to extract impulse-like signals from a correlated non-Gaussian background noise. With properly selected parameters the Adaptive Digital Filtering performs nearly the same as the Kalman Adaptive Filtering. Figure 2 shows the results of Kalman Adaptive Filtering for three channels of data with 960 data points each. This data segment represents a section of complete file. The upper part of each photo is the original data while the lower part is the filtered result. The extracted impulses occur at nearly the same time instants. The only parameters to be selected are AL (the increment) and ITR (the number of iteration). An empirical relationship between AL and ITR is established for each channel as shown in Figure 3. Each point on the curves represents an optimal combination of AL and ITR. Although, the curves are somewhat depending on the data especially on the background noise level, they serve as a useful guideline for the Kalman Adaptive Filtering which is almost nonparametric and is more robust than the Adaptive Digital Filtering. If automatic detection is desired, a correlation or a thresholding operation can be used. Theoretical detection performance, however, is not available.

As the data statistics are generally nonstationary, the parameters of the Adaptive Digital Filtering must be adjusted periodically. No such requirement is needed in Kalman Adaptive Filtering as the final estimate for the previous data point can be used as initial value for the current estimate. Another method to incorporate the time-varying statistics in the Kalman filtering is to use an "on-line" monitoring process to detect the significant change or "jump" in the state and update the state vector accordingly [10]. This nonlinear method however is limited to Gaussian statistics. Based on the above discussion the Kalman Adaptive Filtering method appears to be most suitable for non-Gaussian signal processing. Experimental study has demonstrated good detection performance with the method.

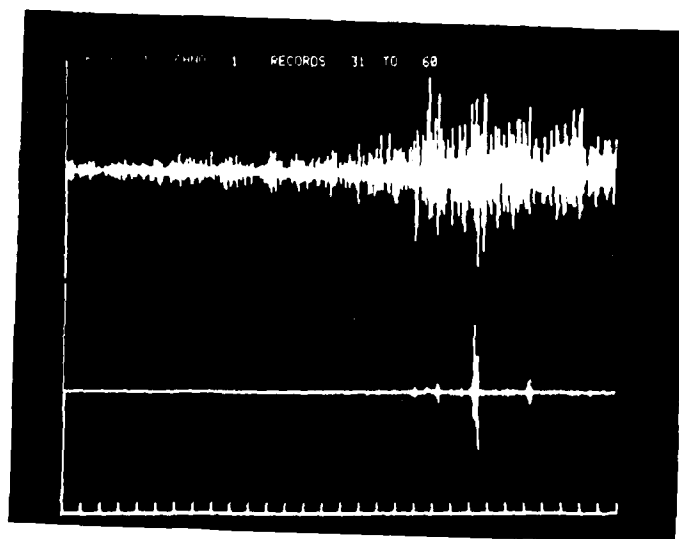


Fig. 2a  
Channel 1 and adaptive  
Kalman filtered data.  
 $AL=0.0038$   
 $ITR=10$

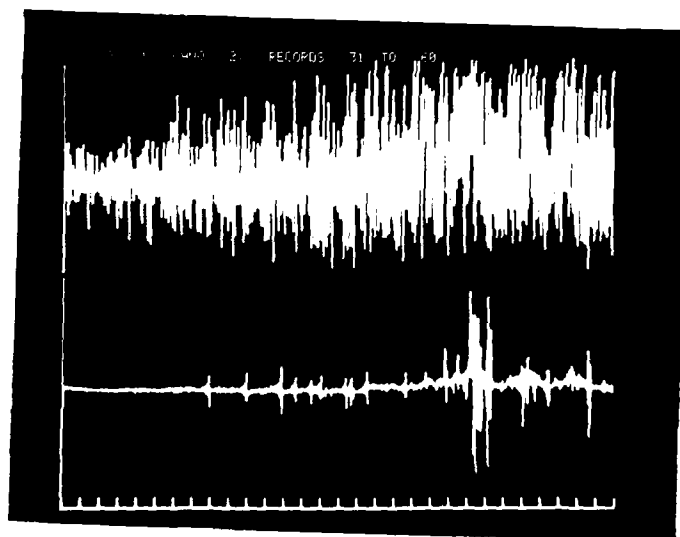


Fig. 2b  
Channel 2 and adaptive  
Kalman filtered data.  
 $AL=0.0028$   
 $ITR=10$

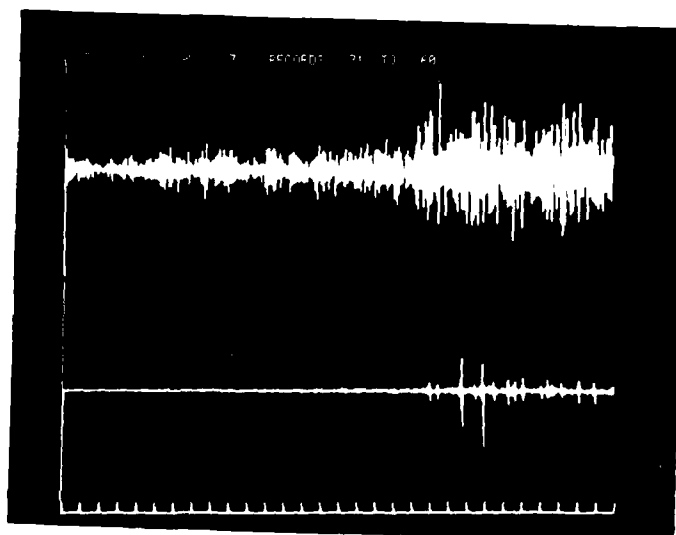


Fig. 2c  
Channel 3 and adaptive  
Kalman filtered data.  
 $AL=0.007$   
 $ITR=10$

A : Channel 3  
 B : Channel 1  
 C : Channel 2

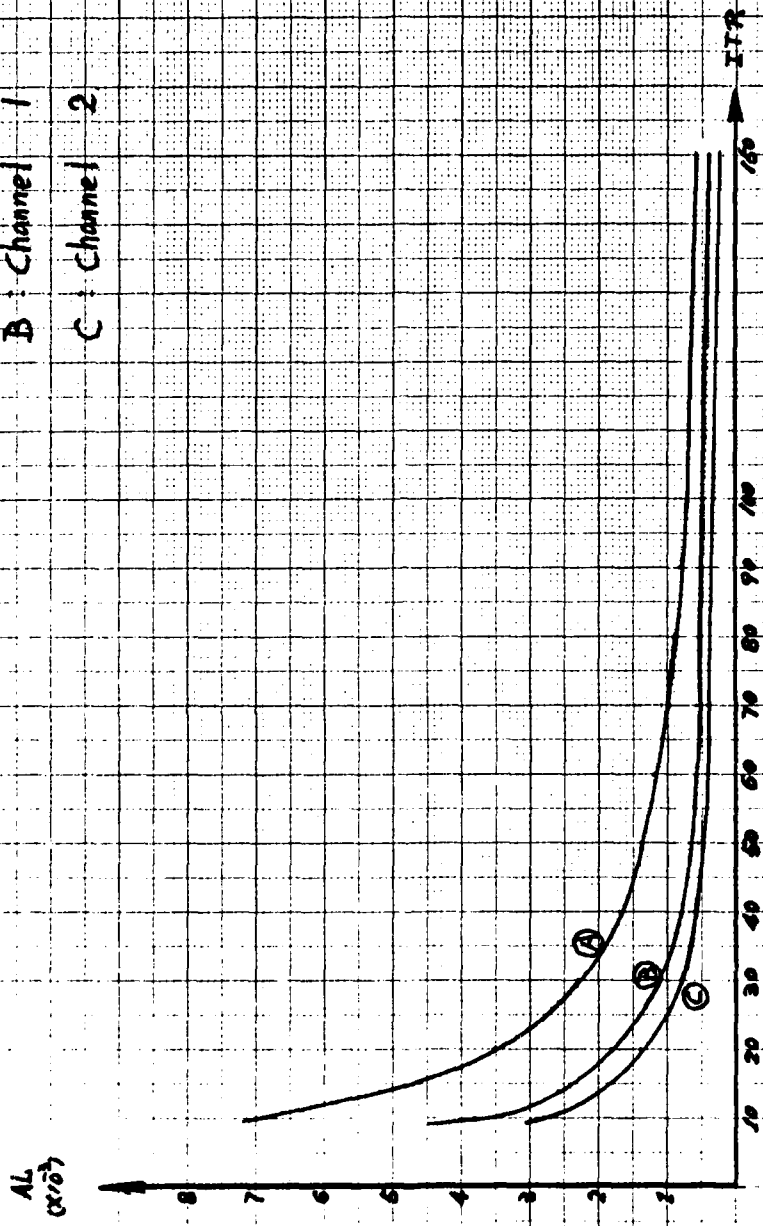


Figure 3 - Relationship between increment (AL) and the number of iterations (ITR) for Kalman filtering of transient signals.  
 Empirical equation:  $AL = f(ITR) = a \exp(-b(ITR))$

### III. Nonlinear Spectral Analysis Techniques

Both the maximum entropy method (MEM) due to Burg and the maximum likelihood method (MLM) for power spectral analysis are considered as non-linear high-resolution spectral analysis techniques suitable for processing transient signals. They both bear a very close relationship to non-linear adaptive array processing techniques [11]. Furthermore the two methods are very closely related [12]. Experimental results have demonstrated that MEM is superior to MLM in spectral resolution (see e.g. [13]). Improvement over the existing Burg's MEM is possible by further reducing the final prediction error with the Fletcher and Powell optimization method as proposed by Fougere [14]. Other algorithms that provide more accurate spectral analysis than the Burg's method have also been proposed (e.g. [15]). The maximum entropy method based on the Fougere's technique has been successfully implemented at the PDP 11/45 minicomputer here at the Southeastern Massachusetts University.

Given  $n$  data points  $x_1, x_2, \dots, x_n$ , we define an  $(m + 1)$  - point prediction error filter  $(1, g_{m1}, g_{m2}, \dots, g_{mm})$  such that the  $k$ th prediction errors are

$$\begin{aligned} \epsilon_{1k} &= \sum_{i=0}^m x_{k+m-i} g_{mi} \\ \epsilon_{2k} &= \sum_{i=0}^m x_{k+1-i} g_{mi} \quad , \quad k = 1, 2, 3, \dots, n-m \end{aligned} \quad (9)$$

where  $g_{m0} = 1$  and  $\epsilon_{1k}$  and  $\epsilon_{2k}$  are the forward and backward prediction errors, respectively. The mean square prediction error, or mean error power, in both time directions is

$$P_m = 0.5(n - m)^{-1} \sum_{s=1}^2 \sum_{k=1}^{n-m} \epsilon_{sk}^2 \quad (10)$$

If the prediction error filter of all orders 1, 2, ..., m are gathered in one matrix  $G_m$  (with the leading "1" suppressed), we may write

$$G_m = \begin{bmatrix} g_{11} & & & \\ g_{21} & g_{22} & & \\ & \vdots & & \\ g_{m1} & g_{m2} & \dots & g_{mm} \end{bmatrix} \quad (11)$$

The off-diagonal elements of  $G_m$  can be determined from the diagonal elements by using the Levinson recursion,

$$g_{jk} = g_{j-1,k} + g_{jj} g_{j-1,j-k} \quad (12)$$

The magnitude of the diagonal elements of  $G_m$  must be less than 1 in order that the prediction error filter be minimum phase. In order to enforce this condition, we can set

$$g_{jj} = U \sin \theta_j \quad (13)$$

where  $\theta_j$  is any real angle and  $U$  is a positive constant slightly less than unity depending on the computer used. For the minicomputer a good choice is  $U = 0.99999$ . Once the computation of the Burg's method is completed the gradient of resulting  $P_m$  with respect to  $\theta$  is determined so that the error minimization procedure can be started. After a number of iterations the filter coefficients can be recomputed from which the power spectrum is determined. Although the original software developed by Fougere is not suitable for the minicomputer, proper modification has made it possible to

use this powerful technique in the minicomputer. Figure 4a shows one cycle of sinewave of 1 Hz at sampling rate of 20 samples per second. Figure 4b shows the Burg's spectrum for the 20 data points. There is spectrum splitting and the spectral peak is shifted to 0.92Hz. The Fougere technique after 10 iterations provides a correctly located peak which is 53 times of the peak in the Burg's spectrum (The vertical scales are not the same in the two photos. Every division in the horizontal scale is 0.5Hz).

It is important to note that very high resolution spectrum can be obtained from the minicomputer using the method described above at only slightly increased computational complexity over the Burg's method. No Gaussian assumption is made in the method. As the method is designed for a small number of samples it is particularly suitable for analyzing the spectrum of the transient signal.

With improved computer hardware the method may become useful for near real-time operations. Each spectral peak can have some physical meaning such as in target detection in clutter. With proper trade-off in speed and accuracy the method can prove to be very useful in non-Gaussian signal processing.

#### IV. Concluding Remarks

Although only two problem areas are considered in this report, non-Gaussian signal processing is a challenging area which requires using knowledge in different disciplines to derive effective solutions. This area also provides an opportunity to integrate or correlate many current signal processing research activities which are developed independently. Continued and strong research effort is needed to pursue new directions and techniques to solve some difficult analytical problems in non-Gaussian signal processing.

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Fig. 4c  
Fougere's Maximum Entropy  
Power Spectrum.



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1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
	AD-A088 558	
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
Study of a Class of Non-Gaussian Signal Processing Problems		Technical Report
		6. PERFORMING ORG. REPORT NUMBER
		SMI-EE-TR-80-6 /
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(s)
C. H. Chen		N00014-79-C-0494 /
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Electrical Engineering Department / Southeastern Massachusetts University North Dartmouth, MA 02747		
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
Statistics and Probability Program Office of Naval Research, Code 436 Arlington, VA 22217		August 12, 1980
		13. NUMBER OF PAGES
		17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release, distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
nonparametric and robust detection Kalman Adaptive Filtering Adaptive Digital Filtering Maximum Entropy Power Spectrum Analysis Weak signal extraction		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
After presenting an overview of important problem areas in non-Gaussian signal processing, the report examines in detail two topics, viz. the non-linear adaptive procedures including the Adaptive Digital Filtering and Kalman Adaptive Filtering for processing non-Gaussian transient signals, and the non-linear spectral analysis techniques for transient signals. A non-linear maximum entropy method is discussed. New computer results on both topics are also presented.		

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